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Conference Paper · August 2014

DOI: 10.13140/2.1.4790.8162

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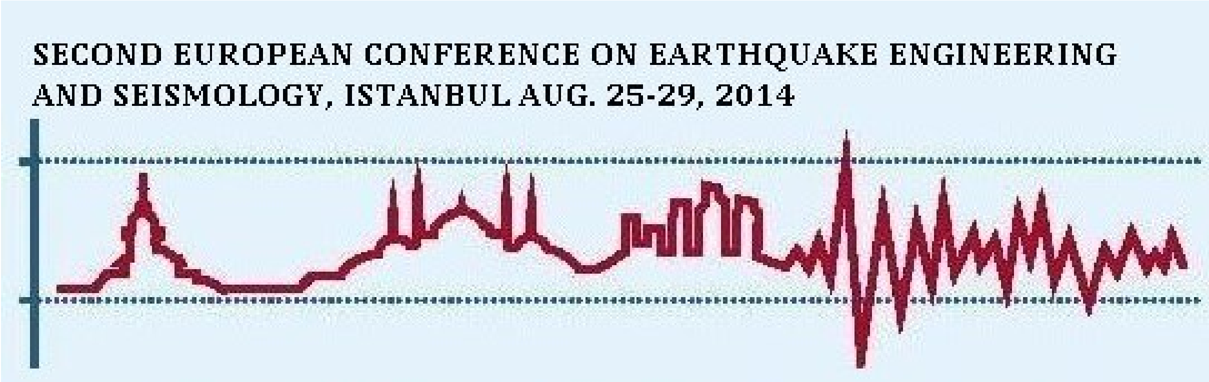

HYBRID LOSS ASSESSMENT CURVE FOR COLOMBIA: A PROSPECTIVE AND A RETROSPECTIVE APPROACH

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ABSTRACT

Countries prone to seismic hazard need to assess the expected risk as a permanent activity in their financial plan; otherwise, they will experience a lack in the information required for the application of disaster risk reduction policies. In this article, a risk assessment methodology is proposed that uses, on the one hand, empiric estimations of loss, based on information available in local disaster data bases, allowing to estimate losses due to small events; on the other hand, it uses probabilistic evaluations to estimate loss for greater or even catastrophic events for which information is not available due the lack of historical data. A "hybrid" loss exceedance curve, which represents the disaster risk in a proper and complete way, is thus determined. This curve merges two components: the corresponding to small and moderate losses, calculated by using an inductive and retrospective analysis, and the corresponding to extreme losses, calculated by using a deductive and prospective analysis.

INTRODUCTION

The assessment of risk due to natural hazards is a task of special concern for the communities settled in hazard prone areas, for the local authorities responsible for the welfare of the population, for the academic community which wants to understand and predict the occurrence of those hazards and also for entities dedicated to improving the living conditions of the communities. The disasters caused by natural hazards undermine the capacities and resources of the affected communities (glazier's fallacy). Furthermore, when those disasters impact recurrently the same settlements, the reconstruction and recovering processes can be interrupted and the consequences of the disasters get deeper. The destruction of assets and the restriction of the capital formation are some of those consequences that also increase the poverty of the members in the affected community and reduce their capacity for adapting and handling future events.

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Intensive risk refers to the possibility of occurring extreme, low frequency hazard events, usually geographically concentrated. It is associated to high intensity hazard events that are able to affect simultaneously an important number of exposed elements, overwhelming the response capacity of the local and even of the national emergency response agencies (Cardona et al. 2008b; Cardona et al. 2012). Extensive risk refers to high frequency events, usually affecting only a few communities each time; in this case, the local and national emergency response mechanisms could be effective when used. Modelling small disasters is a difficult task, especially at country level, due to the large amount of information required and the susceptibility of the results to the local data (like topography or soil mechanics), that is, the results have high variability over small changes of these input data. Even more, the exposure data will also require details only available for big cities but not at rural areas. Nevertheless, the accumulated losses due to extensive, recurrent risk could lead to the exhaustion of the available resources and, thus, to the lack of capacity to absorb future losses and to recover from future events (ERN-AL 2011; UNISDR 2009, 2011, 2013).

The prospective assessment can represent the risk for the low frequency events which, due to their expected magnitude and intensity, can have catastrophic consequences (as they can impact simultaneously large areas and several urban centres). The prospective assessment can be made by methodologies like CAPRA (ERN-AL 2010), which accounts for the uncertainty in the event (location, magnitude and how it manifests itself), the uncertainty in the exposed assets (building response to the event) and the uncertainty in the used models. This assessment of intensive risk is required due to the lack of historical data regarding catastrophic events and the need to anticipate credible future economic losses which, in case of occurrence, could compromise the fiscal sustainability of the affected region or even the country.

The retrospective risk assessment employs an approach similar to the insurance industry, in which data from previous years are statistically processed in order to obtaining a premium for a given sector (e.g. automotive, health, life, home). Finally, using these results, the retrospective loss exceedance curve can be obtained. This curve is of special importance, because it can relate, based on the observed data, economic losses with their expected occurrence frequency; it shows how often an economic loss has occurred or has been surpassed and, if the trends are kept, how often could it be expected or surpassed in the future.

The objective of this article is to describe a new comprehensive methodology for risk assessment in which the accumulated effect of minor and frequent past events is combined with the potential effect of extreme events whose catastrophic impact can have consequences affecting the fiscal and sovereign sustainability of a country. Thus, the proposed approach considers both, the extensive risk which must be retained using reserve funds and reduced with effective vulnerability intervention strategies, as well as the intensive risk, that mainly must be the object of strategies for financial protection and risk transfer (Marulanda et al. 2010; Cardona et al. 2008a). The proposed simplified evaluation of the consequences of small scale disasters, rather than estimating the real or the total value of losses, provides estimates of the minimum cost or minimum impact on the society assets; this minimum cost is the amount which can be expected to be covered by the government due to its fiscal responsibility (if any) with the vulnerable sectors of society, and that has been ignored so far. The effect of insurance policies or risk transfer instruments has not been considered herein.

PROSPECTIVE ASSESSMENT OF RISK

The prospective risk assessment is the evaluation of the consequences of future catastrophic events upon the existing set of exposed assets. These events are modelled based on current knowledge of the movement of the earth's tectonic plates, a detailed definition of the different seismogenic sources (e.g. geometry, location, magnitude recurrence curve) and in some cases the properties of local soil for obtaining a local response (Egozcue et al. 1991). For a probabilistic risk assessment, the definition of the events must include, besides the epicentre and event magnitude, the expected occurrence frequency. The software CRISIS2007 (Ordaz et al. 2007) was therefore selected for the hazard definition; this computer program is part of the CAPRA platform (ERN-AL 2010).

The hazard is characterize by existing studies (AIS 2010, Salgado et al. 2010) and obtained as a catalogue of probable events, in which each event contains its epicentre, magnitude and frequency. The Figure 1 displays the peak ground acceleration (PGA) map for a uniform return period (TR) of 500 years.

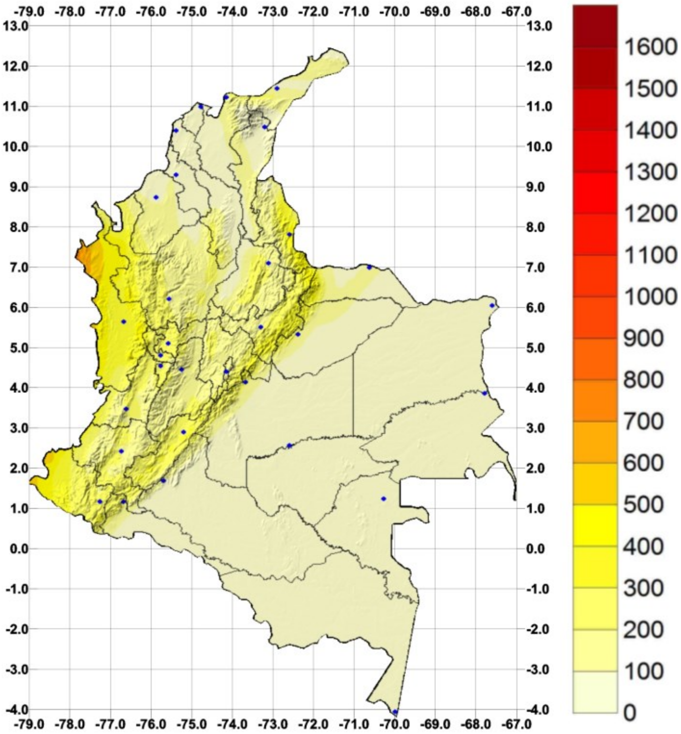

Figure 1. Colombian seismic hazard TR500 – PGA (cm/seg²). Source AIS 2010

After the hazard is defined, the next step consists in defining the set of exposed assets and their expected response (vulnerability) to the selected hazard (Barbat et al. 1998; Barbat et al. 2010; Barbat et al. 2011). Considering the limitations on the existing data (i.e. there is not a building by building country database available that also includes constructive system, foundation, and location) an approximate model or proxy will be defined. The exposed assets proxy will use demographic, construction and macroeconomic indicators and statistics. The vulnerability will be defined using information available in the literature (e.g. HAZUS, Risk-UE, CAPRA) or by using computational models (Lagomarsino et al. 2006; Lantada et al. 2009a, 2009b; Vargas et al. 2013a, 2013b, 2013c).

Table 1 displays the total economic valuation of buildings for different use groups (residential, commercial, industrial, and governmental). The residential use group has been separated in three different subgroups based on the average income in low (L), medium (M) and high (H). Figure 2 shows the exposed value of constructions per municipality.

Table 1. Buildings area and economic valuation

Use group	Construction area [$m^2 x10^3$]	Economic value [US$x10^6$]
Residential L	81,123	17,259
Residential M	297,168	172,987
Residential H	27,700	25,572
Commercial	234,469	129,370
Industry	129,840	114,624
Private Health	263	269

Private Education	27,844	16,603
Public Health	232	181
Public Education	84,111	47,031
Government	4,776.6	2,636
Total	**887,527**	**526,531**

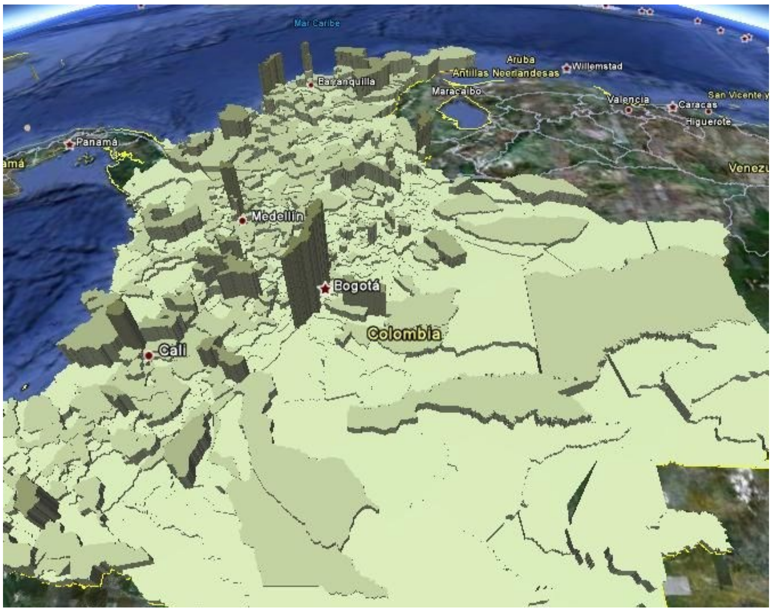

Figure 2. Total exposed value per municipality.

Using the CAPRA platform, it is possible to combine the hazard, the exposed assets database and its vulnerability, allowing the depiction of natural hazard risk by means of the Loss Exceedance Curve (LEC) and of the Average Annual Loss (AAL). The LEC relates a given economic loss (horizontal axis) with the annual frequency (left vertical axis) with which an event could generate a similar or greater loss, as shown on figure 3. The LEC is obtained using the equation

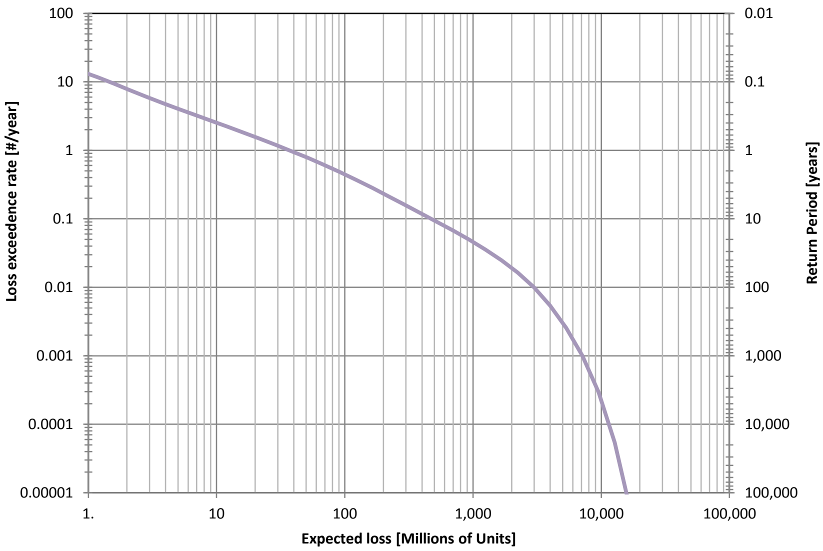

Figure 3. Loss exceedance curve.

$$v(p) = \sum_{i=1}^{events} \Pr(P > p | Event_i) F_A(Event_i) \qquad (1)$$

In this equation, *v(p)* is the exceedance rate of loss, *p*; F_A ($Event_i$) is the annual frequency of occurrence of the $Event_i$; Pr(P > p| $Event_i$) is the probability of the loss to be greater than or equal to *p*, conditioned by the occurrence of the ($Event_i$). The integral over the equation 1 defines the AAL

$$AAL = \int_0^\infty v(p)dp \tag{2}$$

The AAL, could be communicated as the amount of resources that the assessed party (in this case a country) require to save each year in order to recover the losses caused by all the disasters. Even if in some cases the accumulated fund is not enough for covering extreme disasters, this fund will recover over time during future and less catastrophic periods.

As previously stated, the performed analysis accounts only for the country's fiscal portfolio, which was considered to be composed by: low-income population inventory; public assets used for medical and healthcare services; public assets used for educational and cultural services; and public assets and buildings used for administrative services. Table 2 summarizes the probabilistic results obtained for Colombia employing the CAPRA platform; it expresses the results as the probable maximum loss (PML) that can be expected for the exposed portfolio of assets for a specified return period. The PML values depend on the degree of dispersion of the evaluated assets. It should be kept in mind that the values obtained for different return periods correspond to the PML of the entire area and that, when evaluated for a part of this area, they could significantly change because of the level of risk concentration. For fiscal responsibility, the AAL and the PML are obtained at country level. These risk metrics correspond to the losses that the country would have to face due to the potential damage in public and low income population assets which, under the assumptions of this paper, would have to be covered by the government in the case of a major disaster.

Table 2. Probabilistic risk results for Colombia

AAL	
US$ mill.	**‰**
$316	1.8‰

PML		
Return period	**Loss**	
years	**US$ mill.**	**%**
100	$2,976	1.7%
250	$4,417	2.5%
500	$5,655	3.3%
1,000	$7,126	4.1%
1,500	$7,625	4.4%

Figure 4 shows the loss exceedance curve obtained from the probabilistic assessment for Colombia. In this curve, it is possible to see how often an economic loss is expected to be reached or surpassed. For example, it shows that each year an event with losses of at least one million US dollars is expected, and 100 USD millions are lose in average once every two and a half years.

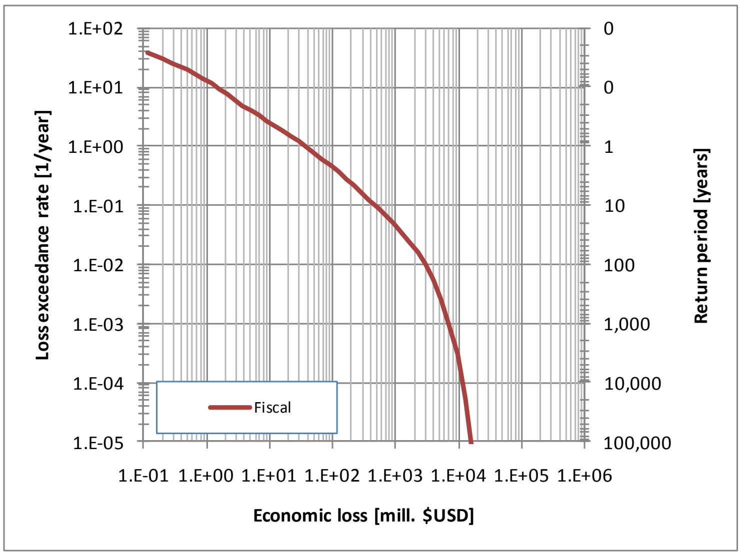

Figure 4. Probabilistic LEC for Colombia – fiscal portfolio

RETROSPECTIVE ASSESSMENT OF RISK

The retrospective assessment of risk corresponds, to the risk associated to small and disperse disasters, which in general relates to the impact caused over small settlements. This approach considers past damages as a predictor of future losses in a methodology similar to the insurance industry, in which data from previous years is used in order of establishing a premium in sectors on which the models are too complex or not reliable enough. In the case of natural hazards, modelling small disasters (in special when evaluating a whole country) is a difficult, if not, impossible task. This is due to several factors like the amount of information required (the whole country) and the susceptibility of the results to small variability on the data (for the hazard, exposure and vulnerability). For the seismic assessment of risk in small areas, the response of the soil is required as well as a more precise definition of the structures and their vulnerability, as it is no longer valid the law of large numbers. This assessment is performed by the means of a statistical analysis over a disaster database like EM-DAT (CRED) or DesInventar (OSSO).

The disaster databases do not store the complete catalogue of historic disasters; instead, they are collecting the more complete set of events (including if possible previous but well documented events). This means that, in the best case, there will be available a sample of events large enough to be studied and from which recommendations could be obtained. Nevertheless, it is important remembering that the information and data stored can have errors and requires a permanent review.

Table 3. Records available per disaster database - Colombia

Database	Period	Records
DesInventar	1914-2011	30,761
EM-DAT	1906-2011	228

Considering the number of available records (Table 3), the spatial resolution (country or municipality) and its public access, the DesInventar disaster database was selected to perform the retrospective assessment of risk. The records available in the database require a process of filtering, which removes records without consequences and records caused by anthropic activities; a process of classification and grouping on which consequences of one event, that are disperse over several records are merge; and finally a process of economic valuation, which allows the assess of economic losses based on the physical damage of the disaster. These processes are described in more detail in the GAR reports (UNISDR 2011 and 2013) and in the corresponding background papers (ERN-AL 2010, CIMNE et al. 2013).

The largest number of records in the DesInventar database corresponds to the sectors of the most vulnerable population and the responsibility of the government upon those sectors, additionaly to their own public assets, will be denominated as fiscal responsibility. It is due to the lack of proper building codes, the lack of enforcement on housing solutions, the lack of land use policies, the lack of prevention works, or because it is required by laws. The damages over the private sector will not be considered as fiscal responsibility, due to the possibility of this sector to assess risk and buy insurance, to access to financial markets and to their own resources.

Therefore, the proposed approach is oriented towards the public sector as a tool to generate concern regarding the losses to the public social infrastructure and to the low-income population sectors that it supports. Those losses, due to their small-scales and high occurrence rate, usually are not covered by the insurance market; therefore the corresponding authorities and the local government need to be aware of this liability. When we couple retrospective risk with the catastrophic one, we obtain a comprehensive view of natural disaster risk which allows its measurement. And, once risk is measured, it is possible to determinate the best policies for its reduction and management.

As the disaster database contains events from other causes besides seismic, it is possible with little or no extra effort to characterize the landscape of small disasters. Figure 5 displays the relative impact that different hazard categories have had over the last 40 or more years and, if the trends are kept, their future impact.

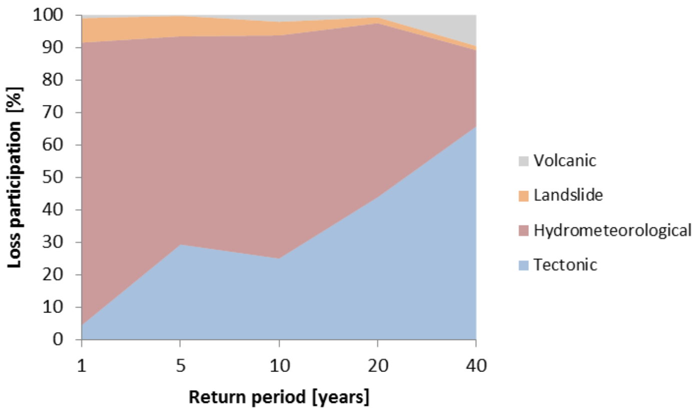

Figure 5. Relative impact of losses per category – Colombia

Finally, as the obtained results represent economic losses, a frequency analysis is possible for each of the categories of natural hazard and for all the categories. Thus, the retrospective or empiric loss exceedance curve is determined. This curve shows the historic frequency with which each loss is reached or exceeded. The loss exceedance curve, LEC, provides the most complete description of risk. It displays the relation between a given loss (usually economic) and the annual rate with which that specific loss will be reached or exceeded. Figure 6 shows a LEC which correlates an expected loss (horizontal axis) with its estimated frequency (left vertical axis). As the frequency is the inverse of the return period, the loss can also be represented as a function of the return period (the right vertical axis).

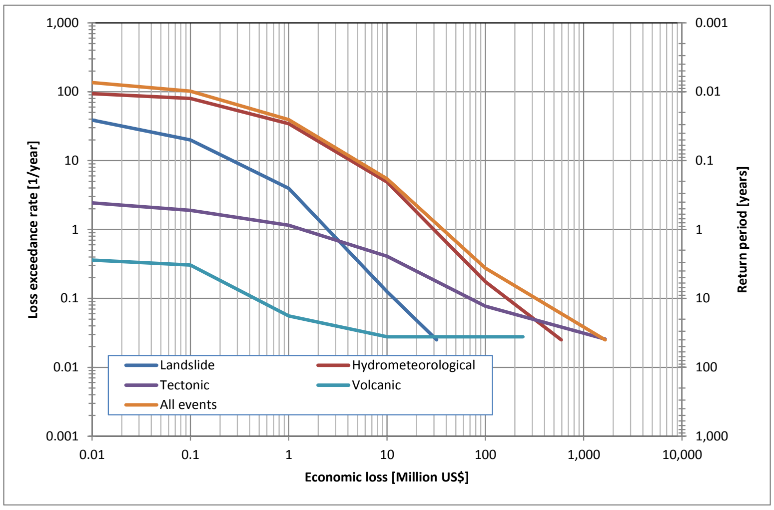

Figure 6. Loss exceedance curves by category for Colombia

The reliability of the analysis is based on the information provided by the disaster databases employed. Thus, there is a permanent need for reviewing and auditing the different database records and their information sources. Other elements, like the chosen variables, the replacement items and their cost, could be improved in country specific assessments.

HYBRID LOSS EXCEEDANCE CURVE

Once the results from the prospective (that accounts for future catastrophic events, which are yet to occur and for which no records exist) and the retrospective (that accounts for small disasters) assessments are obtained, that is, the corresponding loss exceedance curves, LEC, those curves can be coupled in a unique curve. These two segments define the hybrid loss exceedance curve or HLEC (Velasquez et al. 2011 and 2014, UNISDR 2011 and 2013). Figure 7 displays the retrospective and prospective curves obtained in the previous steps and, at the time, same the hybrid curve which is build, in part as the envelope of both curves and in part by joining them by making use of interpolation.

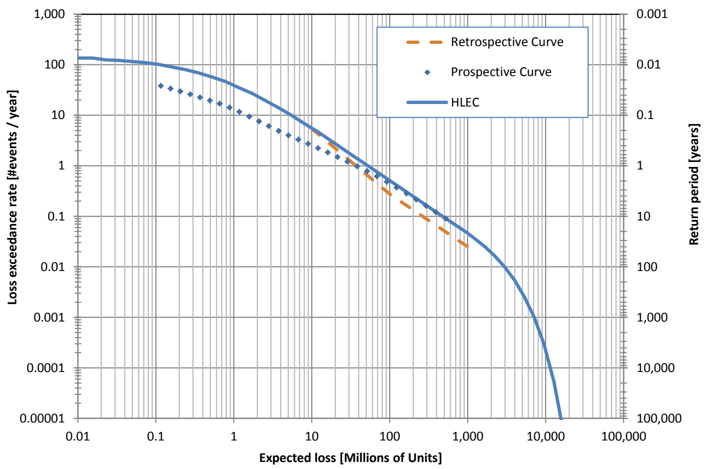

Figure 7. Hybrid loss exceedance curve for Colombia

Other results of the methodology herein proposed, which not necessarily correspond to seismic hazard are displayed in figures 8 to 17. These results were included in the UNISDR Global Assessment Reports (UNISDR 2011, 2013).

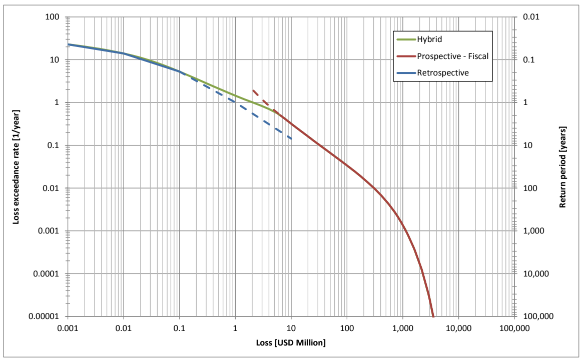

Figure 8 Hybrid LEC for Bolivia

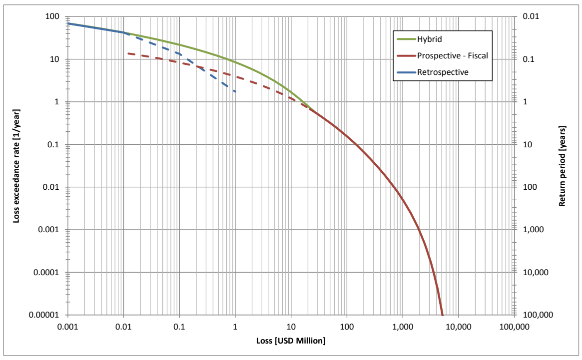

Figure 9 Hybrid LEC for Costa Rica

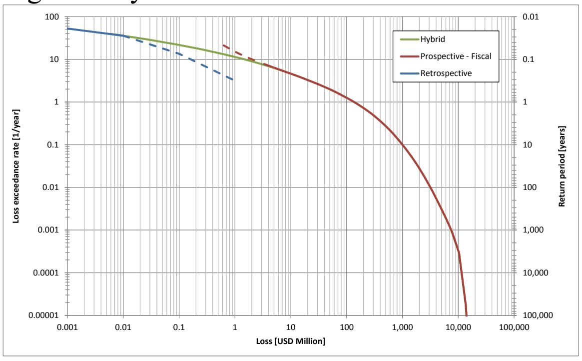

Figure 10 Hybrid LEC for Ecuador

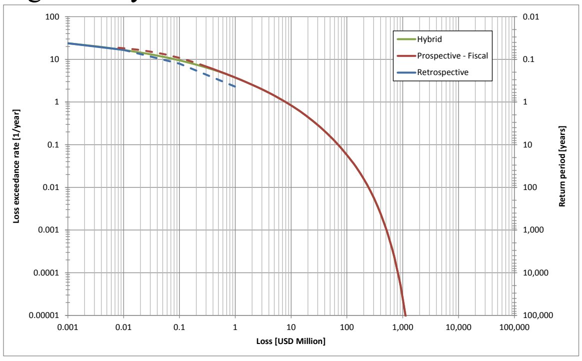

Figure 11 Hybrid LEC for El Salvador

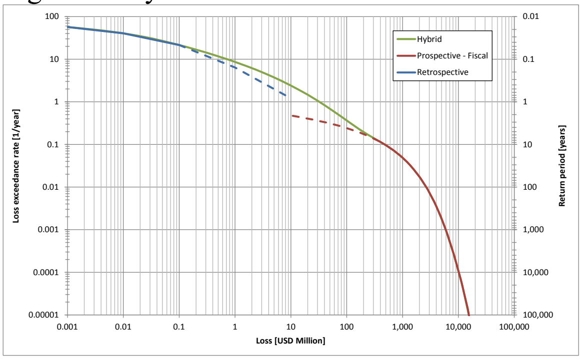

Figure 12 Hybrid LEC for Guatemala

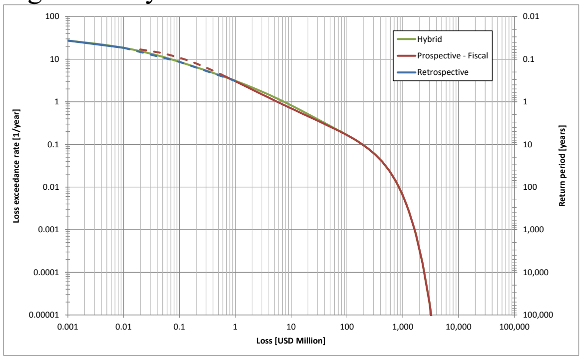

Figure 13 Hybrid LEC for Honduras

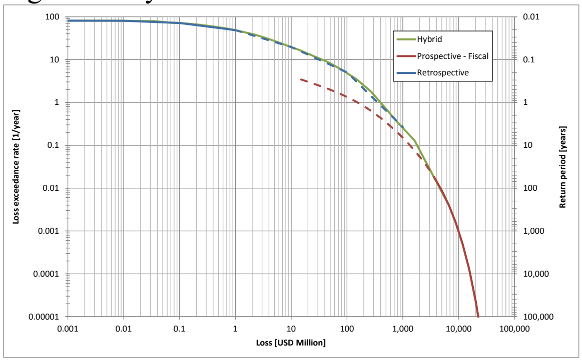

Figure 14 Hybrid LEC for Mexico

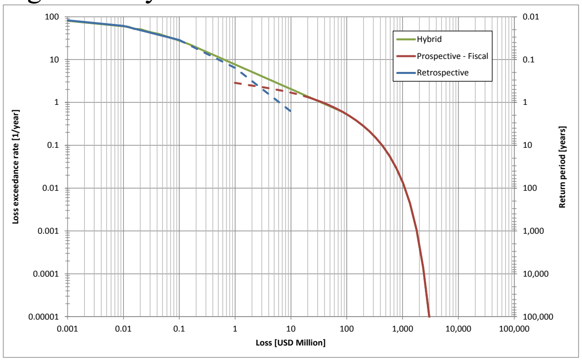

Figure 15 Hybrid LEC for Nepal

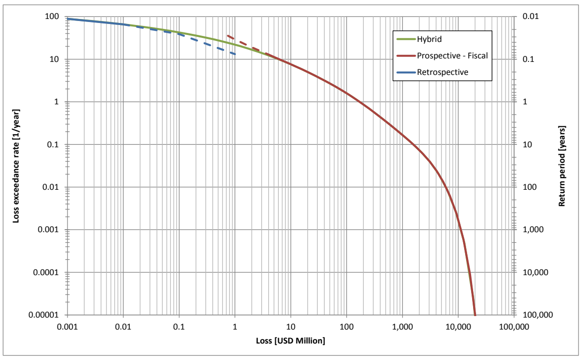

Figure 16 Hybrid LEC for Peru

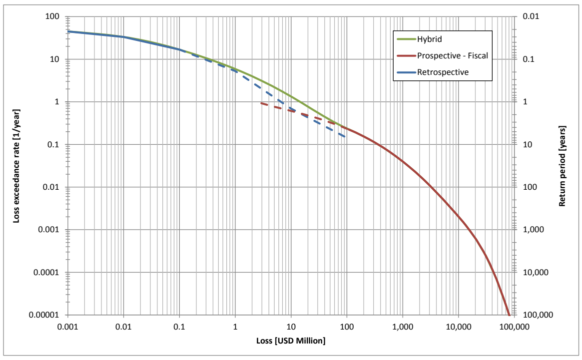

Figure 17 Hybrid LEC for Venezuela

CONCLUSIONS

Employing similitudes with the insurance industry, it is possible to correlate the previous damages and losses to the future risk caused by small disasters (at least at short term). This is particularly helpful when thinking in the complexity of modelling the effect of small disasters over large areas considering, among others, the amount of data, the spatial resolution and the susceptibility of the results.

When the retrospective LEC is combined with the prospective LEC (obtained from a catastrophic risk assessment) it defines the hybrid LEC (Velasquez et al. 2014) providing a more robust and comprehensive profile which can describe, simultaneously, the country's extensive and intensive risk.

The study performed at country level shows that it is indispensable to measure risk retrospectively, with an empirical focus and, at the same time, prospectively, with a probabilistic focus. The lack of procedures to evaluate losses due minor and repetitive events has prevented until now that governments be aware of the enormous losses due to such events and that they retain. The proposed approach and the case studies performed in this article, permit not only to illustrate but also to promote the interest of decision makers towards an effective risk management, based on the complete and multi-hazard risk assessment facilitated by the hybrid loss exceedance curve. The proposed hybrid curve allows capturing aspects which the prospective LEC is not able to consider, avoiding the underestimation of the consequences of small-scale and recurrent events. And, obviously, it is important to have the possibility of estimating expected losses that a country may face perhaps every year and of planning the economical mechanism needed to recover more promptly.

ACKNOWLEDGEMENTS

This work has been sponsored by UNISDR, and has been used as a background paper in the GAR report (UNISDR, 2011, 2013). We want to thank the following local institutions for their help and collaboration: Colombia (INGENIAR, ITEC) and Nepal (NSET). We also thank to the Florida International University for their support in the PhD studies of two of the authors. This work has been also partially sponsored by the European Commission (project DESURBS-FP7-2011-261652). The authors are also grateful for the support of the Ministry of Education and Science of Spain, project "Enfoque integral y probabilista para la evaluación del riesgo sísmico en España, CoPASRE" (CGL2011-29063).

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